IMPROVING POWER CAPABILITY TEST FOR ONE-SIDED TOLERANCES PROCESS WITH MEASUREMENT ERROR

Process capability for one-sided tolerances process

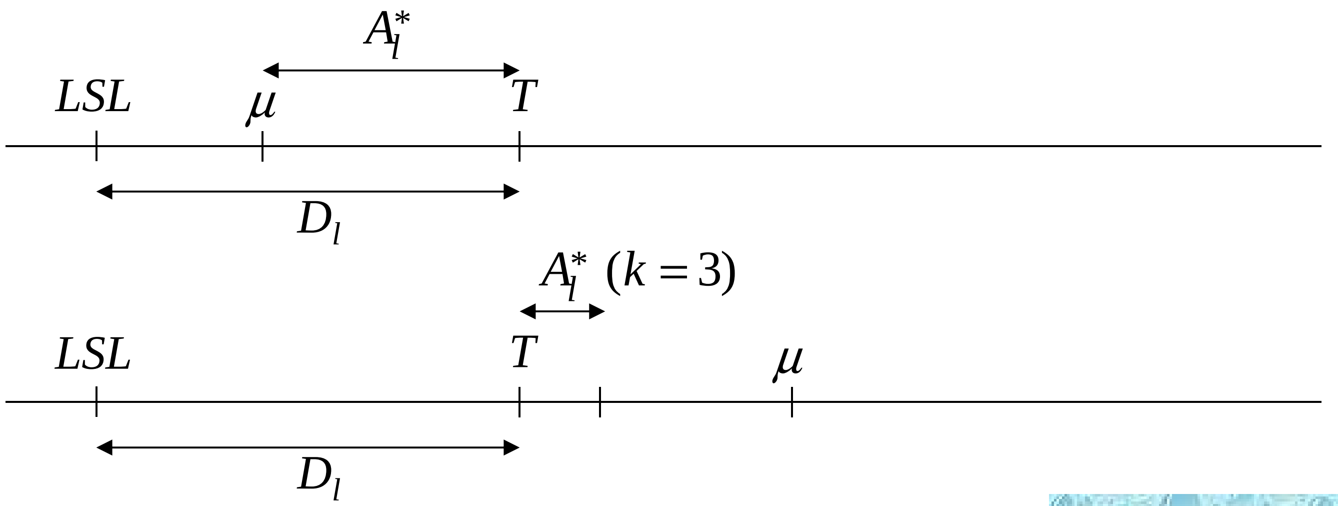
μ, σ, mean and standard deviation of the process

$$C_p^l(u,v) = \frac{D_l - uA_l^*}{3\sqrt{\sigma^2 + vA_l^{*2}}}$$

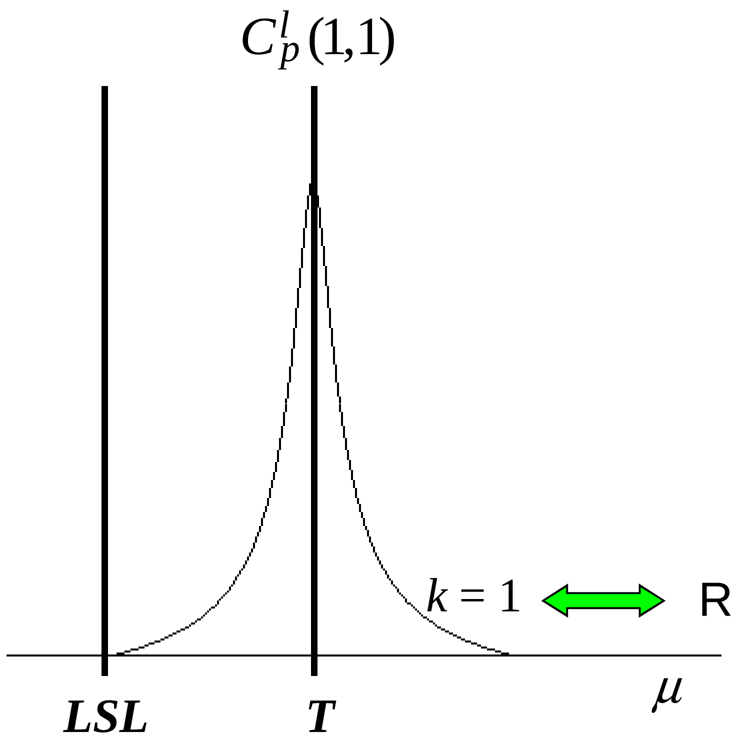
$$D_l = T - LSL$$

$$A_l^* = \max\left(T-\mu, (\mu - T)/k\right) \Longleftrightarrow$$

The risk is considered *k* times less serious to the right of the target T

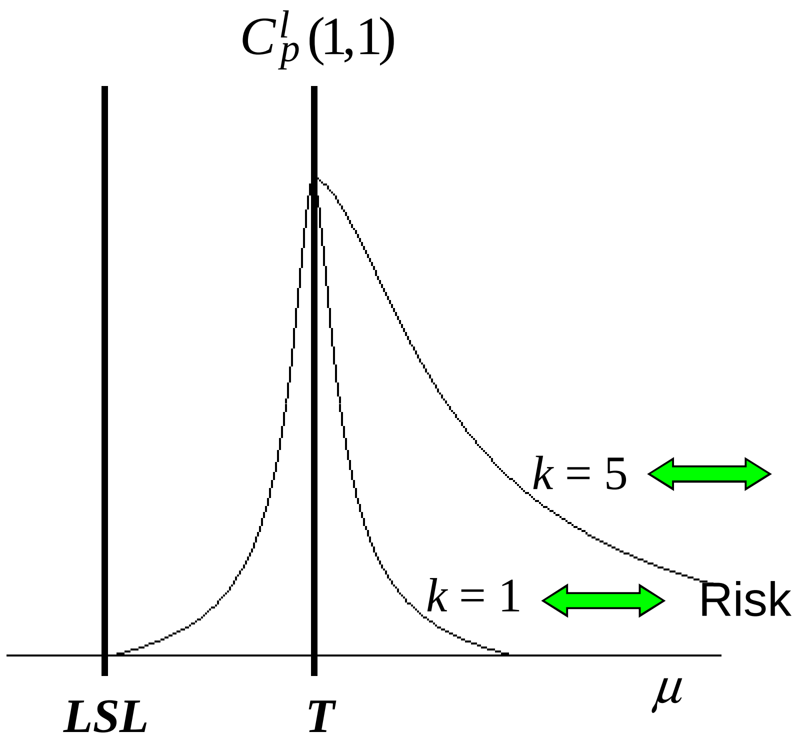


Behavior of $C_p^l(u,v)$



Risk identical to both the left and the right

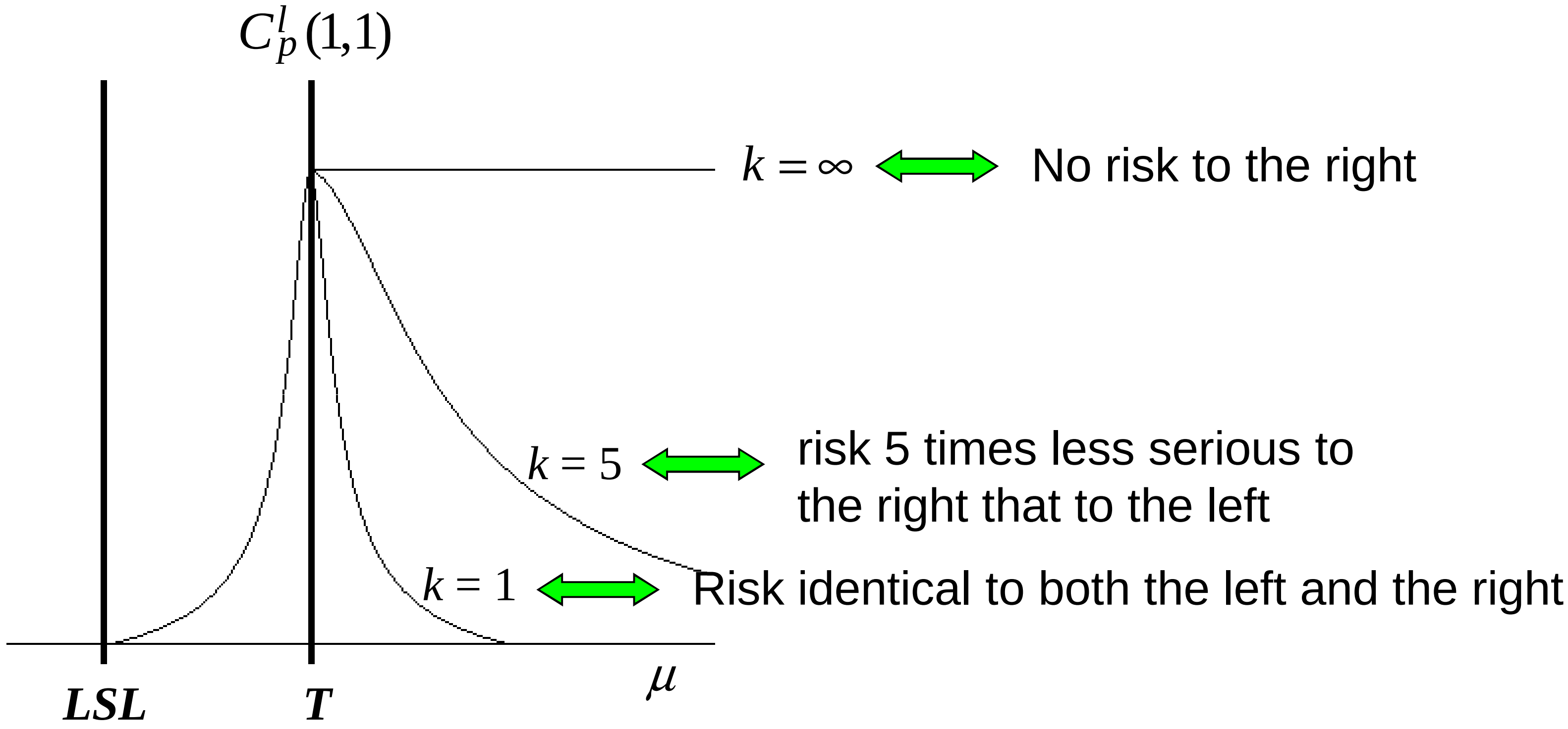
Behavior of $C_p^l(u,v)$



risk 5 times less serious to
the right that to the left

Risk identical to both the left and the right

Behavior of $C_p^l(u,v)$



Measurement errors

Relevant characteristic of the process: $X \sim> N\left(\mu\ ;\ \sigma^2\right)$

Measurement errors: $M \sim> N\left(0\ ;\ \sigma_M^2\right)$

Degree of error contamination (Mittag, 1997): $\tau = \dfrac{\sigma_M}{\sigma}$

the more large is τ, the more important are the errors

Empirical capability of the process

Relevant characteristic of the process : $X \sim> N\left(\mu \; ; \sigma^2\right)$

Measurement errors: $M \sim> N\left(0 \; ; \sigma_M^2\right)$

Observed variable: $G = X + M \sim> N\left(\mu \; ; \sigma_G^2 = \sigma^2 + \sigma_M^2\right)$

Process capability: $$C_p^l(u,v) = \frac{D_l - uA_l^*}{3\sqrt{\sigma^2 + vA_l^{*2}}}$$

Empirical capability of the process : $$C_p^{lG}(u,v) = \frac{D_l - uA_l^*}{3\sqrt{\sigma_G^2 + vA_l^{*2}}}$$

Relation between capability and empirical capability

$$C_p^{lG}(u,v) = \frac{\sqrt{1+v{\xi^l}^2}}{\sqrt{1+\tau^2+v{\xi^l}^2}} C_p^l(u,v)$$

$$\xi^l = \frac{1}{\sigma}\max\left((\mu - T)/k, T-\mu\right) = \frac{A_l^*}{\sigma}$$

No measurement errors $\Leftrightarrow$ $\tau = 0$ $\Leftrightarrow$ $C_p^{lG}(u,v) = C_p^l(u,v)$

Measurement errors: τ increases $\Leftrightarrow$ $C_p^{lG}(u,v)$ decreases

Measurement errors result in an underestimation of the theoretical capability

Estimator $\hat{C}_p^{lG}(u,v)$

Before starting the production control, we collect information on the stabilized process (μ , σ) to construct control charts

r sub-samples $(G_{11}, G_{12}, ..., G_{1n_1})$,..., $(G_{i1}, G_{i2}, ..., G_{in_i})$,..., $(G_{r1}, G_{r2}, ..., G_{rn_r})$

$$\bar{\bar{G}} = \sum_{i=1}^{r} n_i \bar{G}_i / N \qquad S_G^2 = \sum_{i=1}^{r} n_i S_{G_i}^2 / N \qquad N = \sum_{i=1}^{r} n_i$$

$$C_p^{lG}(u,v) = \frac{D_l - uA_l^*}{3\sqrt{\sigma_G^2 + vA_l^{*2}}} \qquad \hat{C}_p^{lG}(u,v) = \frac{D_l - u\hat{A}_l^{*G}}{3\sqrt{S_G^2 + v\hat{A}_l^{*G^2}}}$$

$$A_l^* = \max\left((\mu - T)/k, T - \mu\right) \qquad \hat{A}_l^{*G} = \max\left(\left(\bar{\bar{G}} - T\right)/k, T - \bar{\bar{G}}\right)$$

We obtain the cumulative density function of $\hat{C}_p^{lG}(u,v)$
which depends on $\xi_G = (\mu - T)/\sigma_G$

Test on the process performance

$H_0 : C_p^l(u,v) \le c$ The process is not capable

$H_1 : C_p^l(u,v) > c$ The process is capable

Decision rule

$\hat{c}_p^l(u,v) < c_0 \Leftrightarrow$ We accept H_0 $\Leftrightarrow$ The process is not capable

$\hat{c}_p^l(u,v) > c_0 \Leftrightarrow$ We reject H_0 $\Leftrightarrow$ The process is capable

Determination of the critical value

α = P[reject H_0 / H_0 true]

$$\alpha = P\left(\hat{C}_p^l(u,v) > c_0 \mid C_p^l(u,v) = c\right) = 1 - F_{\hat{C}_p^l(u,v)}(c_0)$$

α and c are given, the distribution of $F_{\hat{C}_p^l(u,v)}$ is known $\Longrightarrow$ c_0

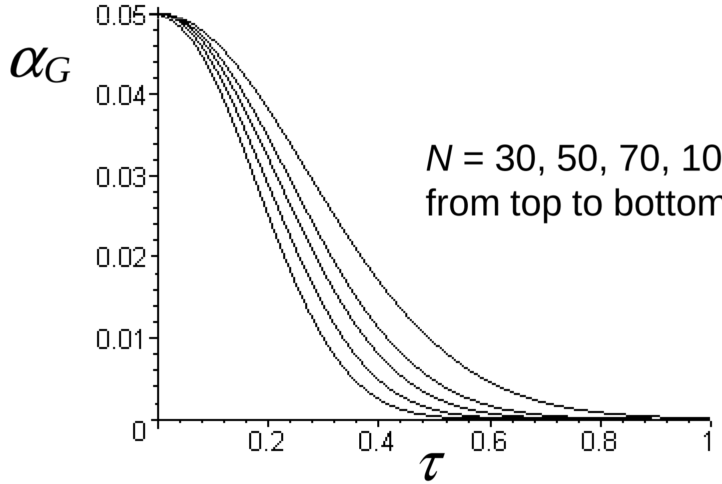
Influence of measurement errors on the level α

In fact it is not $\hat{c}_p^l(u,v)$ that is calculated, but $\hat{c}_p^{lG}(u,v)$

That is calculated with $\hat{C}_p^{lG}(u,v)$ is not α but α_G.

$$\alpha_G = P\left(\hat{C}_p^{lG}(u,v) > c_0 \mid C_p^l(u,v) = c\right)$$

u = 0.5, v = 1.5, k = 3, r = 1,
α = 0.05, c =1.5, $C_p^l(0,0) = 1.5$



N = 30, 50, 70, 100, 150
from top to bottom

The level of the test becoming lower, we tend to accept more easily H_0, thus to conclude the process is not capable even if it is really capable

Influence of measurement errors on the test power

$$\pi\left(C_p^l(u,v)\right) = P\left(reject\ \ H_0 \mid C_p^l(u,v)\right) = P\left(\hat{C}_p^l(u,v) > c_0 \mid C_p^l(u,v)\right)$$

This is not $\hat{c}_p^l(u,v)$ that is calculated but $\hat{c}_p^{lG}(u,v)$

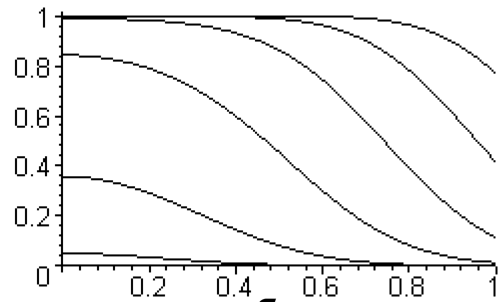
⇒ We calculate $\pi_G\left(C_p^l(u,v)\right)$ and not $\pi\left(C_p^l(u,v)\right)$

$r = 1$, $u = 0.5$, $v = 1.5$, $k = 3$, $r = 1$, $N = 50$, $\alpha = 0.05$, $c = 1.5$

$C_p^l(0,0) = C_p^l(u,v) = 1.5,\ 1.7,\ 1.9,\ 2.1,\ 2.3,\ 2.5$ (*from bottom to top*)

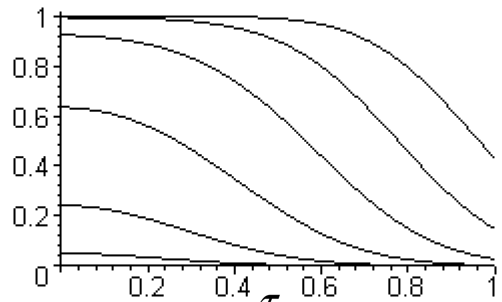
π_G

$\mu > T$



τ

$\mu < T$



τ

The power of the test decreases as the errors increase

How the power of the test can be improved ?

Proposal: modify the critical value c_0 substituting
an adjusted critical value c_0^A as $c_0^A < c_0$

$$\pi_G(\hat{C}_p^{lG}(u,v)) = P\left(\hat{C}_p^{lG}(u,v) > c_0 \mid C_p^l(u,v)\right) \qquad \alpha_G = P\left(\hat{C}_p^{lG}(u,v) > c_0 \mid C_p^l(u,v) = c\right)$$

$$\pi_A(C_p^l(u,v)) = P\left(\hat{C}_p^{lG}(u,v) > c_0^A \mid C_p^l(u,v)\right) \qquad \alpha_A = P\left(\hat{C}_p^{lG}(u,v) > c_0^A \mid C_p^l(u,v) = c\right)$$

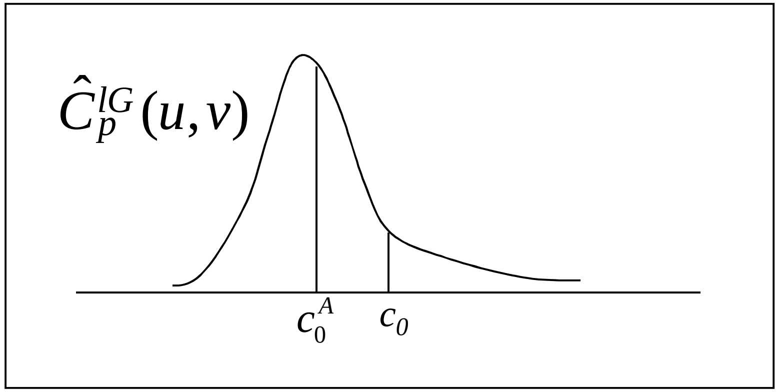
$$c_0^A < c_0$$

$$\Downarrow$$

$$P\left(\hat{C}_p^{lG}(u,v) > c_0^A\right) > P\left(\hat{C}_p^{lG}(u,v) > c_0\right)$$

$$\Downarrow$$

$$\pi_A > \pi_G$$



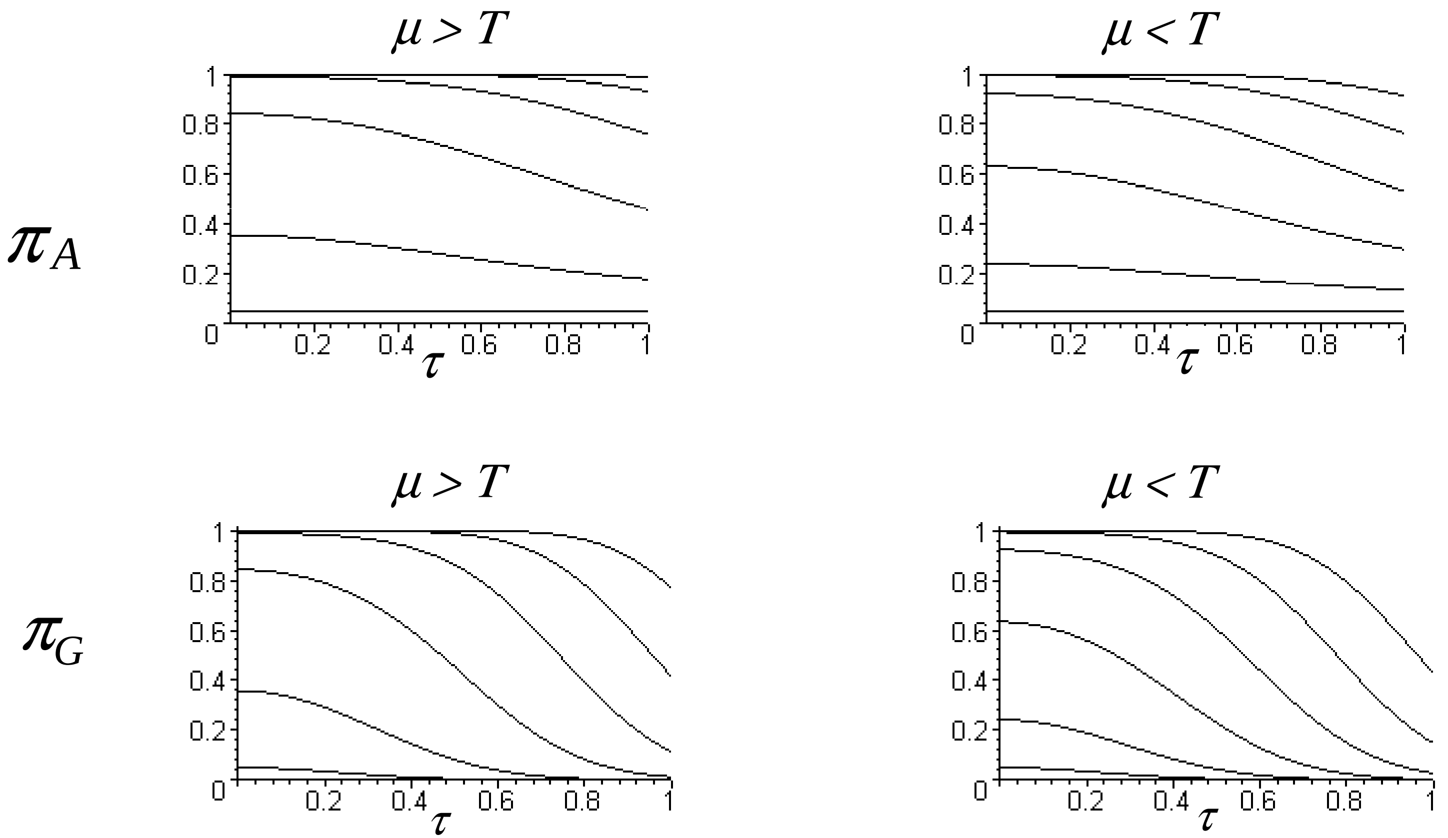
but also $\alpha_A > \alpha_G$

So that the level of the test is the initial level, we set $\alpha_A = \alpha$

Comparisons of curves π_A and π_G

$r = 1,\ u = 0.5,\ v = 1.5,\ k = 3,\ r = 1,\ N = 50\ ,\ \alpha = 0.05,\ c = 1.5$

$C_p^l(0,0) = C_p^l(u,v) = 1.5,\ 1.7,\ 1.9,\ 2.1,\ 2.3,\ 2.5$ (*from bottom to top*)



Application example

Manufactury of nougat within the company Chabert et Guillot in Montélimar

Lot of 10,000 nougat bars are sold to wholesalers for a nominal weight of 200g each

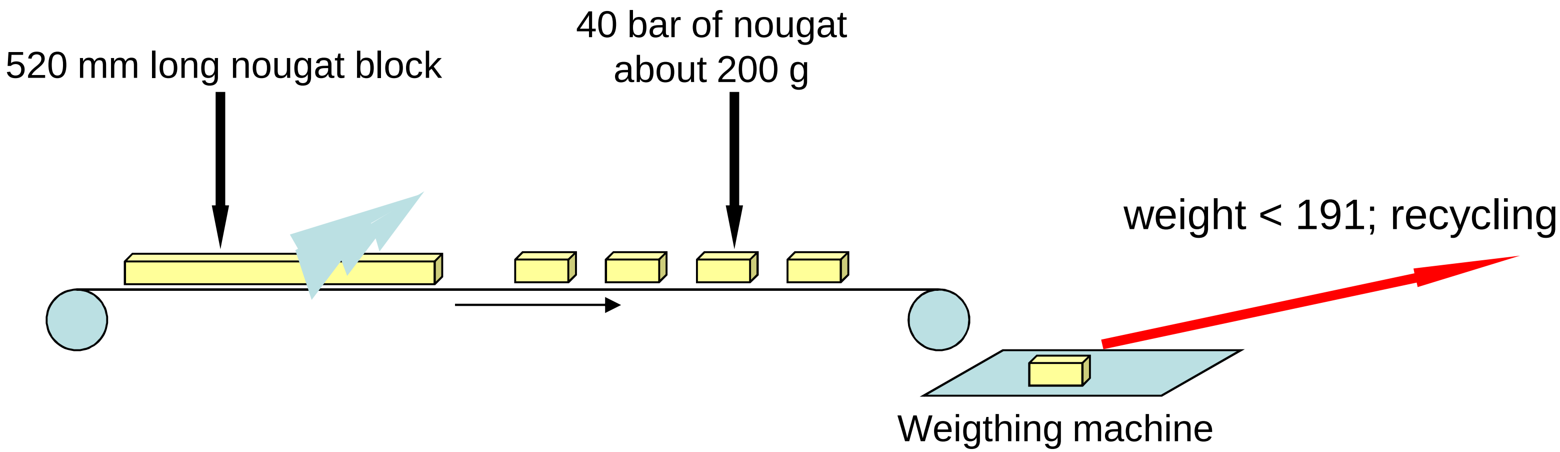
French legislation imposes

- No bar should weigh less than 182 g
- It should not be more than 2% of bars between 182 g and 191 g
- The average weight must be at least 200 g

⇨ LSL = 191, T = 212

No upper limit but overdose

⇨ A shift to the right is considered to be three times less serious than a left shift (k = 3)



Choose of the capability index $C_p^l(u,v)$

The pair (u,v) is choosen according to

M, maximum proportion of acceptable non-conforming,

and K, maximum acceptable deviation

For the company capable process $\Longleftrightarrow$ $\begin{cases} C_p^l(u,v) \geq 1 \\ \mu > 200 \\ M \text{ minimum} \end{cases}$ $\Longrightarrow$ $K = (T-\mu)D_l = 57\%$

pairs (u,v) where K is equal to 57% when $C_p^l(u,v) = 1$

(u,v)	(0.1,0.3)	(0.4,0.2)	(0.8,0.1)
K	57%	57%	57%
M in ppm	6037	3645	1681

The company chooses $C_p^l(0.8, 0.1)$

To show the interest to take into account measurement errors we take $C_p^l(0.1, 0.3)$

20 blocks were taken

Due to a systematic deformation at the beginning and the end of the block, only 36 of 40 bars are weighed

Sample number	1	2	3	4	5	6	7
$\overline{g}_i$	208.506	210.116	208.797	209.814	210.463	209.562	209.897
s_i	4.712	3.648	3.264	3.437	4.418	4.518	4.225
Sample number	8	9	10	11	12	13	14
$\overline{g}_i$	209.553	209.286	209.577	210.851	210.168	211.011	211.018
s_i	4.632	4.825	4.654	4.365	4.153	4.419	5.170
Sample number	15	16	17	18	19	20	
$\overline{g}_i$	209.464	210.554	209.582	209.776	211.002	210.882	
s_i	4.709	4.695	4.258	4.943	4.834	3.946	

$r = 20, n = 36, N = 720$ $\quad \overline{\overline{G}} = 209.994$ $\quad S_G = 4.418$ $\quad \hat{\xi}_G = -0.454$ $\quad \hat{c}_p^{lG}(0.1, 0.3) = 1.523$

Calculation of the adjusted critical value: τ need to know

A R&R study of Repetability ans Reproductibility gives $\tau = 0.18$

$$\left.\begin{array}{l} c = 1 \\ \text{level } \alpha = 0.05 \\ \hat{\xi}_G = -0.454 \end{array}\right\} \Longrightarrow c_0^A = 1.052 \quad < \hat{c}_p^{lG}(0.1, 0.3) = 1.523$$

$\Longrightarrow$ The process is capable

Note: fictitious example

Suppose that a sample gives $\bar{\bar{G}} = 209.100$ and $S_G = 6.388$

$\Longrightarrow \hat{c}_p^{lG}(0.1, 0.3) = 1.049$

The choice of $\bar{\bar{G}}$ and S_G was made so we have the same value $\hat{\xi}_G = -0.454$

$\tau = 0 \Longrightarrow c_0 = 1.058 \quad > \hat{c}_p^{lG}(0.1, 0.3) = 1.049 \Longrightarrow$ Not capable process

$\tau = 0.18 \Longrightarrow c_0^A = 1.043 \quad < \hat{c}_p^{lG}(0.1, 0.3) = 1.049 \Longrightarrow$ Capable process

Ignoring measurement errors can lead to reject a process capable

THANK YOU FOR YOUR ATTENTION