

Observations, sensitivity and Bayesian inference in QMRA

- **Background of the problem: dealing with uncertainty and probabilistic inference in RA.**
- **Two example problems where overall sensitivity of surveillance matters.**
- **Discussion.**



Engineering example: Falcon HTV-2 hypersonic aircraft

- First tested in wind tunnel conditions and computer simulations
 - "It's time to conduct another flight test to validate our assumptions and gain further insight into extremely high Mach regimes that we cannot fully replicate on the ground."
 - "We wouldn't know exactly what to expect based solely on the snapshots provided in ground testing. Only flight testing reveals the harsh and uncertain reality."

Air Force Maj. Chris Schulz / CBSNEWS 11.Aug.2011.

Remarks from Falcon HTV-2:

- **Experimental conditions do not match real situation.**
- **Simulations may largely rely on assumptions.**
- **Data from actual flight conditions needed!**

Compare with QMRA on food production chains:

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Compare with QMRA on food production chains:

- Experimental conditions do not match real situation.
- Simulations may largely rely on assumptions.
- Data from actual *production chain* needed!

Idea (*not new, but still...*) of probabilistic evidence synthesis:

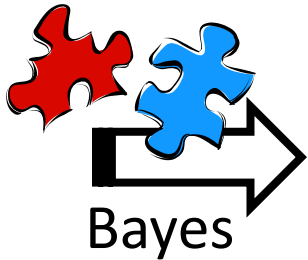
- **Experimental conditions** can provide prior information.
- **Assumptions** can describe other prior information.
- Data from actual ***production chain*** is direct evidence.



$P(\theta)$



$P(X | \theta)$



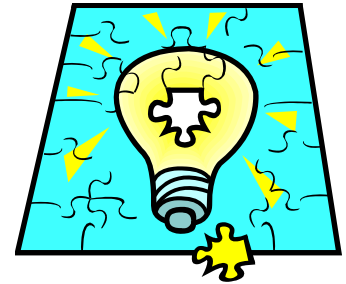
$$P(\theta | X) = \frac{P(X | \theta)P(\theta)}{\int_{\Theta} P(X | \theta)P(\theta)d\theta}$$



Another example: controlling FMD outbreak

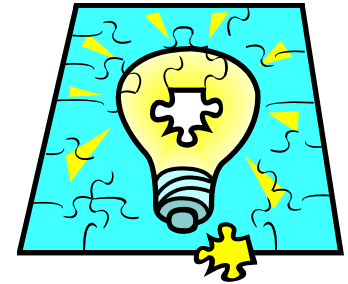
- "The models' veterinary **assumptions did not fit** with either field or experimental reality and represented a different (wholly theoretical) virus".
- "The use of **non-validated models** as predictive tools to guide policy during an epidemic of FMD remains highly questionable, especially given the **imprecision of the available data** and the **complex nature** of the biology of FMD virus"
 - Mansley LM, Donaldson AI, Thrusfield MV, Honhold N: Destructive tension: mathematics versus experience – the progress and control of the 2001 foot and mouth disease epidemic in Great Britain. Rev. sci. tech. Off. int. Epiz., 2011, 30 (2), 483-498.

Underestimating uncertainty? Can these problems be avoided ?

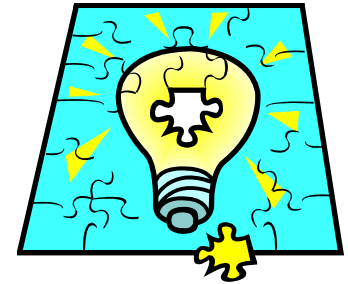


- Experiments are difficult, or impossible, with a real production chain.
 - Costs are much too high, consequences irreversible.
 - Can be forbidden by law.

Underestimating uncertainty? Can these problems be avoided ?



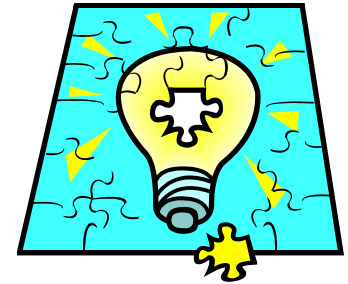
- Experiments are difficult, or impossible, with a real production chain.
- Purely experimental conditions are simplified versions of reality – can we extrapolate?
 - Infection experiments possible with some inoculated animals under test conditions. E.g. hens in cages.
 - Other contamination experiments under 'similar' conditions.
 - Only limited number of factors can be controlled.



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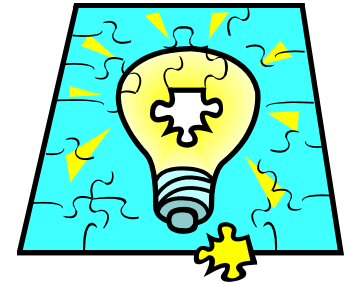
- Experiments are difficult, or impossible, with a real production chain.
- Purely experimental conditions are simplified versions of reality – can we extrapolate?
- Real life data from other production systems usually context dependent, systems not identical – can we borrow estimates?
 - Can data from one slaughterhouse describe those in all EU MSs?
 - Is any study from one country informative of others?
 - Meta analysis methods?

Underestimating uncertainty? Can these problems be avoided ?



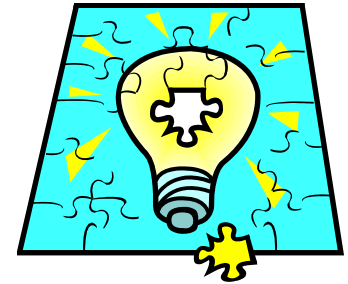
- Experiments are difficult, or impossible, with a real production chain.
- Purely experimental conditions are simplified versions of reality – can we extrapolate?
- Real life data from other production systems usually context dependent, systems not identical – can we borrow estimates?
- Computer simulations with Monte Carlo methods based on assumptions – not (only) estimates from actual field data.
 - Assuming e.g. that all practices over the production chain are done according to adopted guidelines, protocols and laws.

Underestimating uncertainty? Can these problems be avoided ?



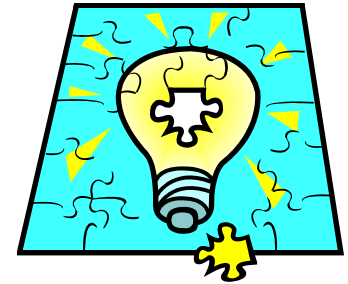
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- Computer simulations with Monte Carlo methods based on assumptions – not (only) estimates from actual field data.
- Mechanistic modelling aims to replicate reality in detail but will introduce lots of uncertain parameters.
 - Is every detail needed?
 - Criteria for relevant scope of the model in relation to what is known?
→ model uncertainty! Model assessment.

Underestimating uncertainty? Can these problems be avoided ?



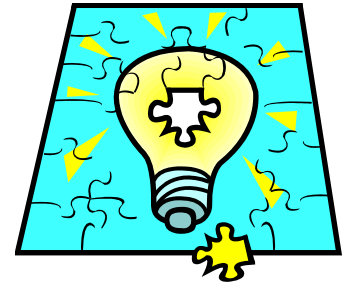
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- Simple statistical models do not utilize relevant prior knowledge.
 - ‘Modeling the data’ vs. ‘modeling the problem’?

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Underestimating uncertainty? Can these problems be avoided ?

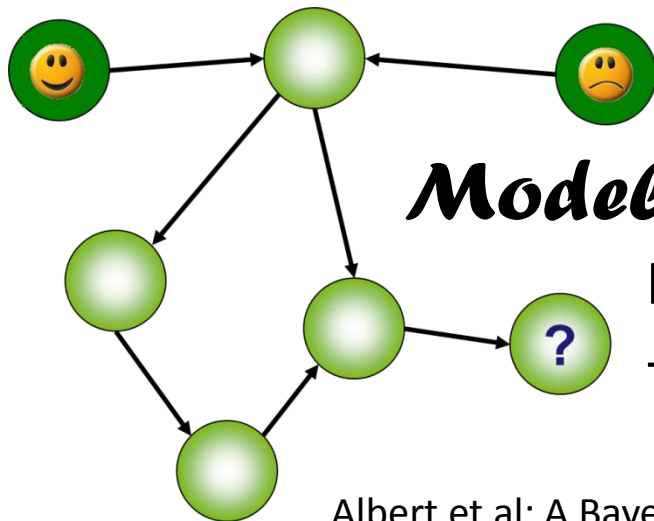


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- Simple statistical models do not utilize relevant prior knowledge.
- **Models may not be valid, but do we have valid data either?**
 - Good quality does not mean it's valid for RA-question.

The Quest for More Unifying Treatment of Uncertainty

Need to find a link between different uncertainties (aleatory / epistemic)

- Directed Acyclic Graph (DAG) of the whole model clarifies a bit
- Information on one part of the model is connected to others (Bayes)



Modeling evidence synthesis

Is it feasible?

- Some examples exist

→ More examples needed.

Albert et al: A Bayesian evidence synthesis for estimating campylobacteriosis prevalence, Risk Analysis, 31(7), 1141-55. 2011.

Surveillance data from food production chain

- Data *will constrain* what can be inferred...
- Sensitivity of the surveillance method is part of the problem.

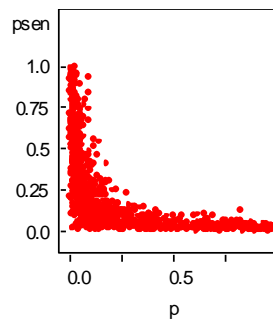
➤ We may need to model both p & p_{sen} .

➤ Identifiability?

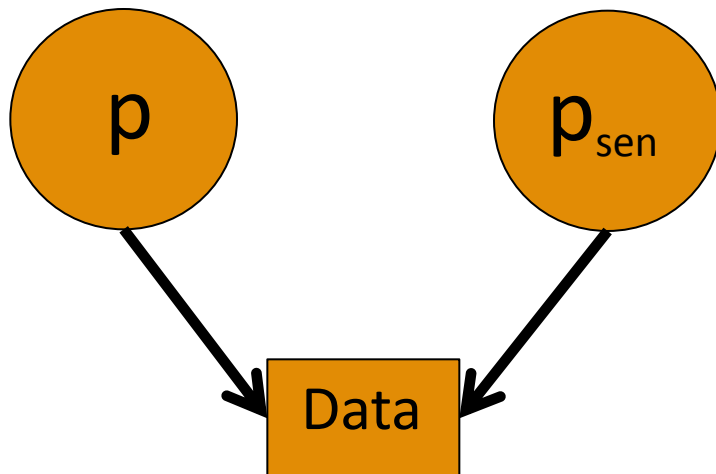
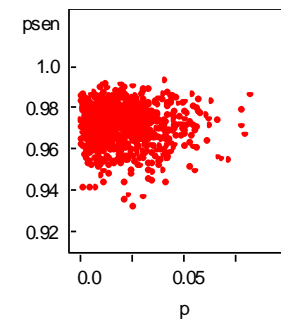
Assume $N=100$ tested, $X=1$ positive:

Two Priors:

$P(p_{\text{sen}}) = U(0,1)$

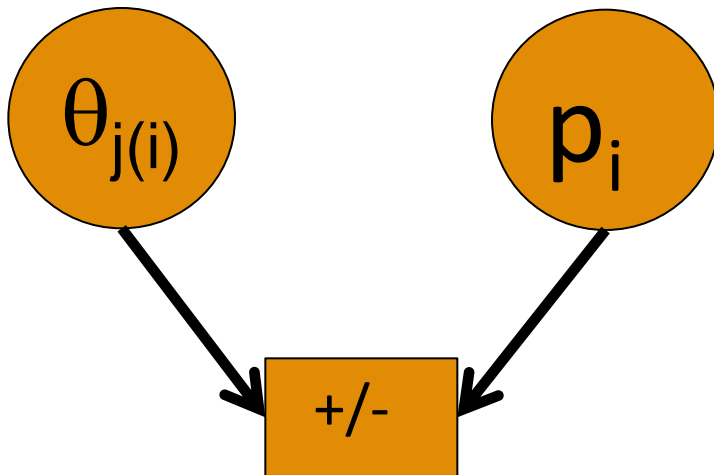


$E(p_{\text{sen}})=0.97, SD(p_{\text{sen}})=0.01$



Salmonella surveillance: detection of infected cattle herds

- In each geographical area A_i
 - True infection prevalence $\theta_{j(i)}$
 - p_i = overall sensitivity of detection, for the year.
 - #positives detected = Y_i & # cattle herds = N_i known.
tested herds = Z_i is unknown in 'small' areas (regionally known).



- p_i = overall sensitivity, depends on

- $\alpha = P(\text{tested} \mid \text{clinical symptoms})$

Global
parameter

- $\beta_i = P(\text{tested} \mid \text{no clinical symptoms})$

Regional
parameter

- k = Number of animals pooled (if pooled)

- p_w = Within herd prevalence

- p_f = Laboratory sensitivity of faecal test

- $Y_i \sim \text{Binomial}(N_i, P(\text{detection}))$

$$P(\text{detection}) = \theta_{j(i)} (\alpha + \beta_i - \alpha\beta_i) p_i$$

$$p_i = P(CS \setminus NCS \mid \text{tested}) p_f$$

$$+ P(NCS \setminus CS \mid \text{tested}) \sum_{k=1}^K (1 - (1 - p_w)^k) p_f P(k)$$

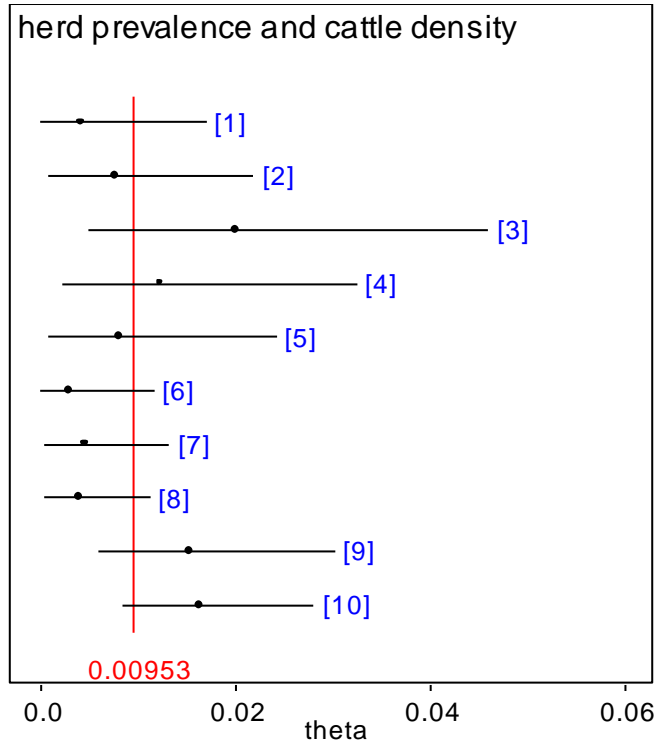
$$+ P(CS \cap NCS \mid \text{tested}) \left[1 - (1 - p_f) \left(1 - \sum_{k=1}^K (1 - (1 - p_w)^k) p_f P(k) \right) \right]$$

Laboratory sensitivity p_f is only one small part of this!

Information about each parameter exploited,
WinBUGS model \rightarrow posterior distribution.

After some MCMC simulations

<1.2
Animals
per km²
land area
>11.6



(10% of municipalities in each class [1]-[10])

22

agrostat2012, 29.2.
jukka.ranta@evira.fi

```

mode[]
for(i in 1:437){ # foreach municipality in 1999 in Finland
  cattledens[i] <- cattle[i]/area[i] # cattle density
  herdsinf[i] ~ dbin(pinf[i], herds[i]) # number of infected herds
  pinf[i] <- expression1[i] + expression2[i]
  expression1[i] <- p[1]*step(1.18739-cattledens[i]) + p[2]*step(1.89658-cattledens[i])*(1-step(1.18739-
cattledens[i])) +
  p[3]*step(2.52852-cattledens[i])*(1-step(1.89658-cattledens[i])) + p[4]*step(3.37003-cattledens[i])*(1-step(2.52852-
cattledens[i])) + p[5]*step(4.40459-cattledens[i])*(1-step(3.37003-cattledens[i]))
  expression2[i] <- p[6]*step(5.41454-cattledens[i])*(1-step(4.40459-cattledens[i])) + p[7]*step(6.59755-
cattledens[i])*(1-step(5.41454-cattledens[i])) + p[8]*step(8.29316-cattledens[i])*(1-step(6.59755-
cattledens[i])) + p[9]*step(11.6191-cattledens[i])*(1-step(8.29316-cattledens[i])) + p[10]*step(cattledens[i]-11.6191)
  hprev a[i] <- herdsinf[i]/herds[i] # herd prevalence
  herdsinf sel[i] ~ dbin(pselinf[i], herdsinf[i]) # selected infected herds
  pselinf[i] <- pncs[i] + alpha - pncs[i]*alpha # chance of selection for testing
  herdspos[i] ~ dbin(hsens[i], herdsinf sel[i]) # detected positive herds (given as data)

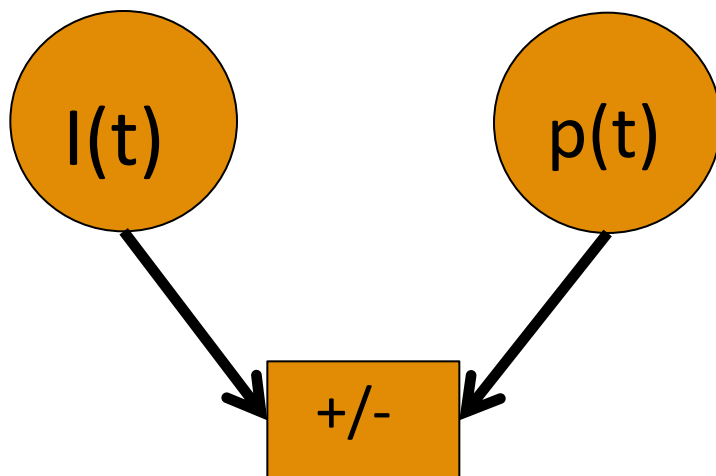
  denom[i] <- alpha*alpha*pncs[i] + pncs[i]
  hsens[i] <- ((alpha*alpha*pncs[i])/denom[i])*labsens +
  ((pncs[i]-alpha*pncs[i])/denom[i])*ncssens +
  ((alpha*pncs[i])/denom[i])*(1-(1-labsens)*(1-ncssens)) # herd level test sensitivity
  riskcattle[i] <- meancattle[i]*herdsinf[i] # estimated number of cattle in infected herds.
  # probability of being tested (independent of salmonella status):
  pncs[i] <- equals(province[i], 1)*beta[5] + equals(province[i], 2)*beta[6] +
  equals(province[i], 3)*beta[1] + equals(province[i], 4)*beta[11] +
  equals(province[i], 5)*beta[3] + equals(province[i], 6)*beta[2] +
  equals(province[i], 7)*beta[7] + equals(province[i], 8)*beta[10] +
  equals(province[i], 9)*beta[4] + equals(province[i], 10)*beta[9] +
  equals(province[i], 11)*beta[12] + equals(province[i], 12)*beta[8]
}
#### sensitivity of pooled samples: #####
ncssens <- sum(A[i]) # NCS sensitivity
for(i in 1:10){ # equal probabilities for selecting 1-10 animals in a joint sample
  A[iii] <- (0.01)*((1-pow(1-pwithin, iii))) * labsens
}
for(i in 11:15){ # equal probabilities for selecting 11-15 animals in a joint sample
  A[iiii] <- (0.03)*((1-pow(1-pwithin, iiii))) * labsens
}
for(i in 16:20){ # equal probabilities for selecting 16-20 animals in a joint sample
  A[iiiij] <- (0.15)*((1-pow(1-pwithin, iiiij))) * labsens
}
hinf sel <- sum(herdsinf sel[i]) # total number of infected herds that become selected for testing
meanhsens <- mean(hsens[i]) # mean herd sensitivity (overall herd sensitivity)
##### prior distributions: #####
lymphsens ~ dbeta(279.7, 652.6333) # mean=0.3, std=0.015
labsens ~ dbeta(74.31556, 6.462222) # prior for laboratory sensitivity
pwithin ~ dunif(0, 1) # prior for expected within herd prevalence
# prior for chance to become CS tested while infected:
alpha ~ dbeta(3.057145, 6.86865)
# prior for chance to become NCS tested while infected:
beta[12] ~ dunif(beta[11], ncs[12])
beta[11] ~ dunif(beta[10], ncs[11])
beta[10] ~ dunif(beta[9], ncs[10])
beta[9] ~ dunif(beta[8], ncs[9])
beta[8] ~ dunif(beta[7], ncs[8])
beta[7] ~ dunif(beta[6], ncs[7])
beta[6] ~ dunif(beta[5], ncs[6])
beta[5] ~ dunif(beta[4], ncs[5])
beta[4] ~ dunif(beta[3], ncs[4])
beta[3] ~ dunif(beta[2], ncs[3])
beta[2] ~ dunif(0, ncs[2])
beta[1] ~ dunif(0, 1) # dummy variable, not used.
beta1 <- 0 # this is beta for A land
ncs[1] <- 0.00001 # dummy variable, not used.
for(e in 2:12){
  ncs[e] <- ncstests[e]/herdsprovinced[e] ...and continues
}

```

Salmonella surveillance: detection of infected laying flocks



- $I(t)$ = true infection status at time t : $I(t) = 1$ or 0 .
- t = time since beginning of laying period.
- $p(t)$ = overall sensitivity of testing at time t .
- Testing results D_1, D_2, D_3, \dots at times t_1, t_2, t_3, \dots



$I(t)$ = hidden Markov process,
Intensities λ and μ .

$p(t)$ can change due to
within flock epidemic .

First approach had assumed
 $p(t) = p$ for all times t .

Hidden Markov process:

Transition probabilities for a flock: $0 \rightarrow 1$, and $1 \rightarrow 0$

$$p_{01} = P(I_t = 1 | I_0 = 0, \lambda, \mu) = \frac{\lambda}{\lambda + \mu} - \frac{\lambda}{\lambda + \mu} \exp(-(\lambda + \mu)t)$$

$$p_{10} = P(I_t = 1 | I_0 = 0, \lambda, \mu) = \frac{\lambda}{\lambda + \mu} + \frac{\mu}{\lambda + \mu} \exp(-(\lambda + \mu)t)$$

Solve the probability of hidden positive, at age t .

This can be written recursively (Nagelkerke *et al.*, 1990):

$$\rho_t = \begin{cases} \frac{(1-p)[(1-p_{01}-p_{10})\rho_{t-1} + p_{01}]}{1-p[(1-p_{01}-p_{10})\rho_{t-1} + p_{01}]}, & t > 1 \\ \frac{(1-p)P(I_1 = 1)}{(1-p)P(I_1 = 1) + P(I_1 = 0)}, & t = 1 \end{cases}$$

Here: $\rho_t = P(I_t = 1 \mid D_1, \dots, D_t)$, depends on the history.

If $D_t=1$, then we know $I_t=1$, but typically $D_1=0, \dots, D_t=0$

- Since salmonella is a rare thing (in Finland):
 λ is likely to be small
- Since salmonella is hard to clear out from a flock:
 μ is likely to be small

→ Expected time to event can exceed
expected life time!

What is known of sensitivity $p(t)$?

- EFSA report: *"the rate of transmission of Salmonella within a flock determines the change in within-flock prevalence, which, in turn determines when a colonised flock can be detected"*.
- If flock is very recently infected, detection not likely.
- After 2-4 weeks since infection, detection almost sure. (Within flock prevalence gets $> 5\%$).
- After some longer time, detection may be less optimal, (intermittent shedding, immunity, adaptation, other effects?)

What is known of sensitivity $p(t)$?

- EFSA report: *"most hens stop shedding the bacteria after approximately three weeks"*.
 - AND: *"over time the number of organisms excreted by infected birds, and as a consequence, the within-flock prevalence, may decrease"*.
- To simplify, this could be described **by a function of duration** of infection in the flock.
 - Duration depends on time of infection. This is unknown parameter τ_0 .

• Sensitivity as a function of duration $d = t - \tau_0$?

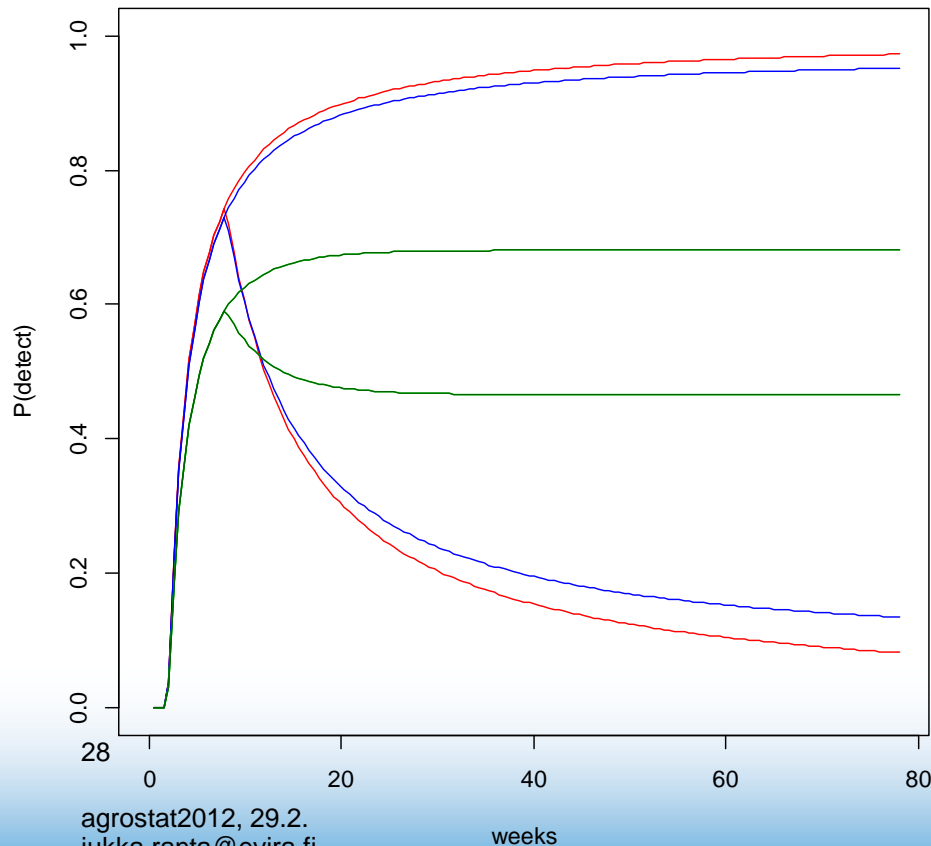
- E.g. simple function:

$$P(\oplus_t | \tau_0, I_t = 1) = p(d) = \begin{cases} 0 & , \text{if } d < d_1 \\ p^* & , \text{if } d_1 < d < d_2 \\ 0 & , \text{if } d > d_2 \end{cases}$$

- To get sensitivity at time t (age of flock), we need to integrate over unknown τ_0 .

$$P(\oplus_t | I_t = 1) = \int_0^t P(\oplus_t | \tau_0, I_t = 1) \pi(\tau_0 | I_t = 1) d\tau_0$$

$$\Rightarrow \frac{p^* e^{-\mu t}}{1 - e^{-\mu t}} (e^{\mu \max\{t-d_1, 0\}} - e^{\mu \max\{t-d_2, 0\}})$$



$$\leftarrow p^* = 1, \mu = 0.1, 1, 10, \\ d_1 = 2/52, d_2 = 8/52, d_2 = \infty$$

• Sensitivity as a function of duration $d = t - \tau_0$?

– But which function?

$$P(\oplus_t | \tau_0, I_t = 1) = p(d) = \begin{cases} 0 & , \text{if } d < d_1 \\ p^* & , \text{if } d_1 < d < d_2 \\ 0 & , \text{if } d > d_2 \end{cases} \quad \text{Step function}$$

$$P(\oplus_t | \tau_0, I_t = 1) = p(d) = p^* e^{-0.5(d-a)^2 / \sigma^2} \quad \text{Gaussian curve}$$

$$P(\oplus_t | \tau_0, I_t = 1) = p(d) = p^* (1 - e^{-ad}) \quad \text{Exponential increasing}$$

$$P(\oplus_t | \tau_0, I_t = 1) = p(d) = p^* (1 - e^{-ad}) e^{-a \max\{d - d^{**}\}} \quad \text{Exponential increasing, \& decreasing}$$

Desideratum:

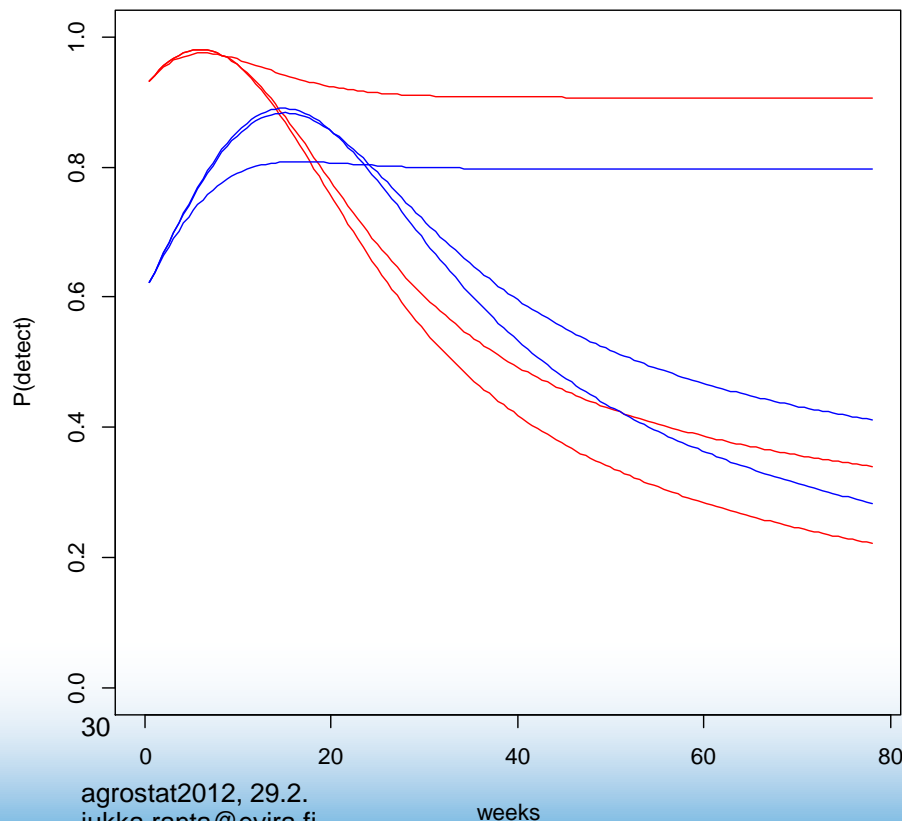
- Should reflect assumptions
- Should be simple (for ease of integration)
- As few parameters as possible
- Parameters with meaningful interpretation (to be elicited)
- To be fitted to experimental data, if ever available

- Sensitivity as a function of duration $d = t - \tau_0$?

- For example, Gaussian function gives:

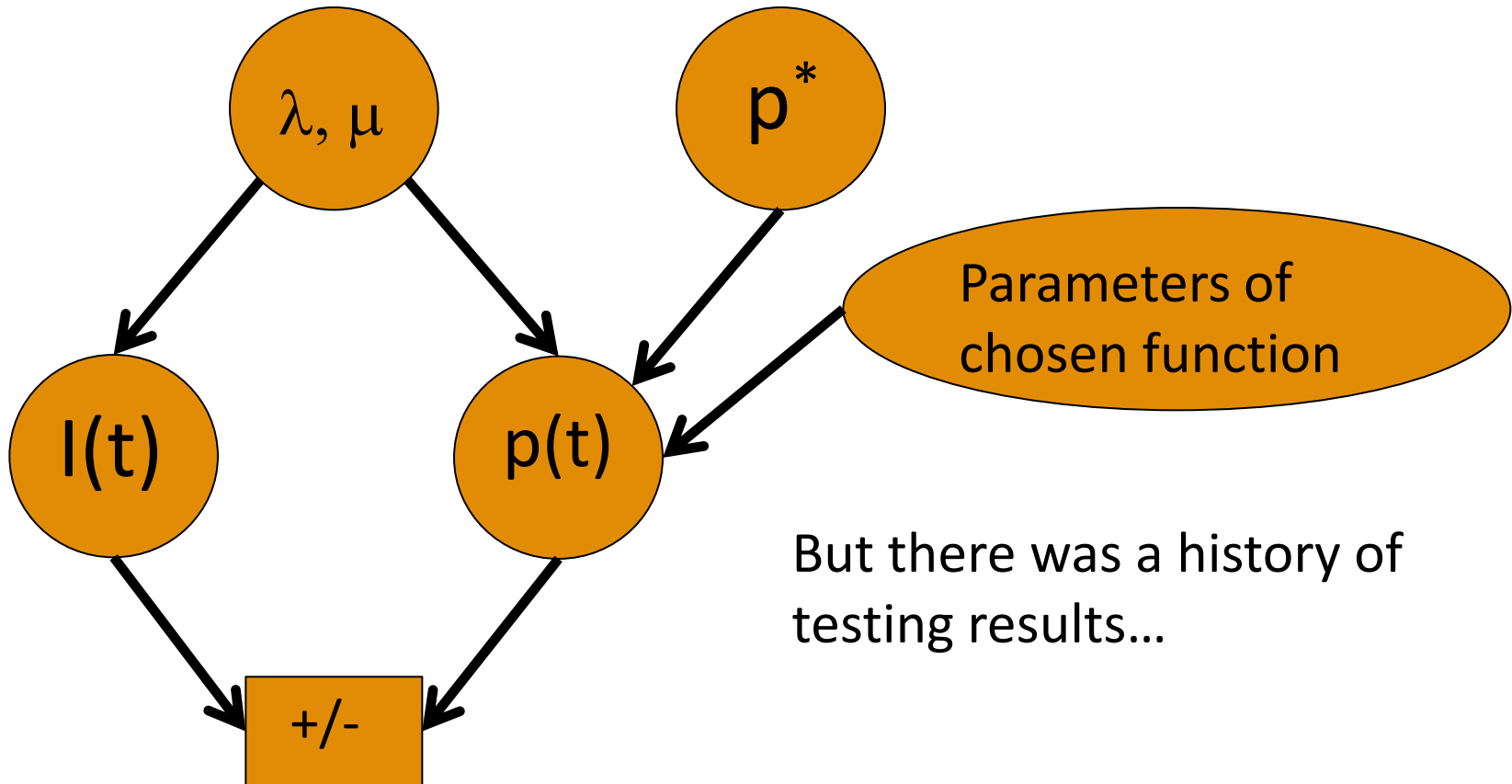
$$P(\oplus_t | I_t = 1) =$$

$$\frac{p^* \mu}{1 - e^{-\mu t}} e^{-0.5(a^2 - (a - \mu \sigma^2)^2) / \sigma^2} \sqrt{2\pi} \sigma [\Phi((t - a + \mu \sigma^2) / \sigma) - \Phi((-a + \mu \sigma^2) / \sigma)]$$



$p^* = 1$, $a=4/52$, $a=10/52$
 $\sigma=10/52$,
 $\mu = 0.1, 1, 10$

- DAG of previous models for hidden process and sensitivity, with one testing result only:



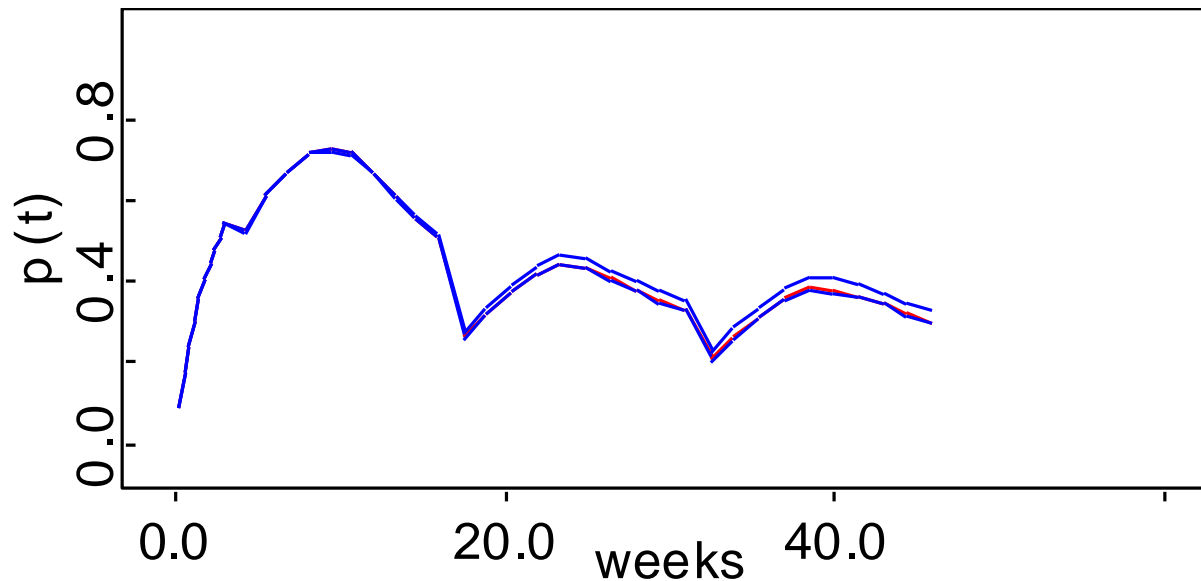
- Past testing results provide evidence about where τ_0 probably was, given that $I(t)=1$.
 - The history is necessarily a series of negatives (*positive flocks are eliminated*).
 - To compute $p(t)$ at a time point t between 1st and 2nd testing times t_1 and t_2 , e.g. with Gaussian function:
- $$P(\tau_0 | I_t = 1) \propto \begin{cases} e^{-(t-\tau_0)\mu} (1 - p^* e^{-0.5(t_1-\tau_0-a)^2/\sigma^2}) & , \text{ if } \tau_0 \in [0, t_1] \\ e^{-(t-\tau_0)\mu} & , \text{ if } \tau_0 \in [t_1, t], t < t_2 \end{cases}$$
- More testing times \rightarrow more steps!
 - Normalizing constant by numerical integration.

- Finally, solving again:

$$P(\oplus_t \mid I_t = 1) = \int_0^t P(\oplus_t \mid \tau_0, I_t = 1) \pi(\tau_0 \mid I_t = 1) d\tau_0$$

- But this also becomes numerical, because of a more complicated $\pi(\tau_0 \mid I_t = 1)$.
- But OpenBUGS has a procedure for this:
`result <- integral(F(),start,end,accuracy)`
- Becomes eventually slow, but better than simulating τ_0 for every flock.

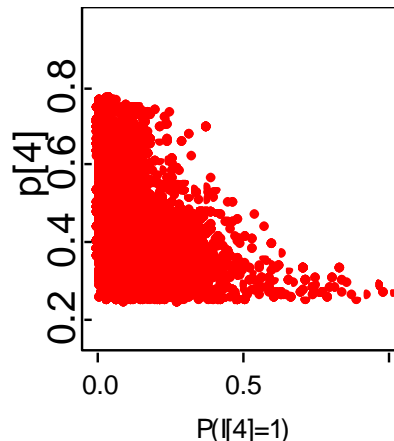
- **Test results:** compute posterior, for example with $p^* = 0.9$, and with exponential increase & decrease with $a = -\log(0.05)/(4/52)$, $d^{**} = 8/52$



- **Test results:** posterior probability of true prevalence at 4 testing times:

	mean	sd	val2.5pc	median	val97.5pc
$p(I[1]=1)$	0.1579 %	0.143	0.01861	0.1146	0.6258
$p(I[2]=1)$	0.1312 %	0.08776	0.02076	0.1098	0.353
$p(I[3]=1)$	0.1332 %	0.1013	0.01676	0.1079	0.3773
$p(I[4]=1)$	0.1559 %	0.1313	0.01395	0.1174	0.486

- Finally, estimate true prevalence over laying period to quantify risk of production → further used in RA.

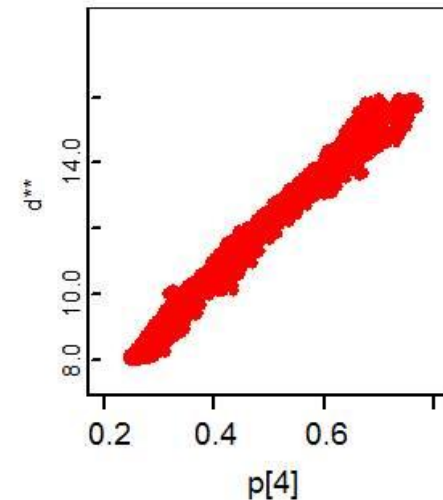
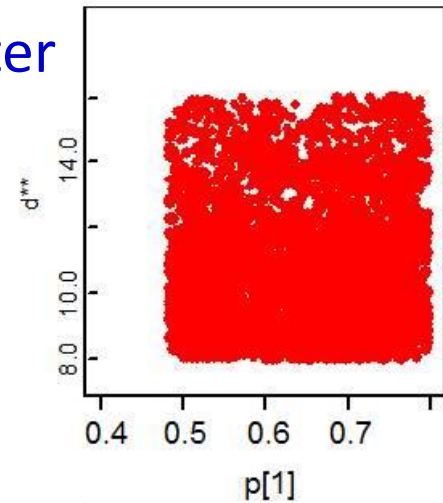
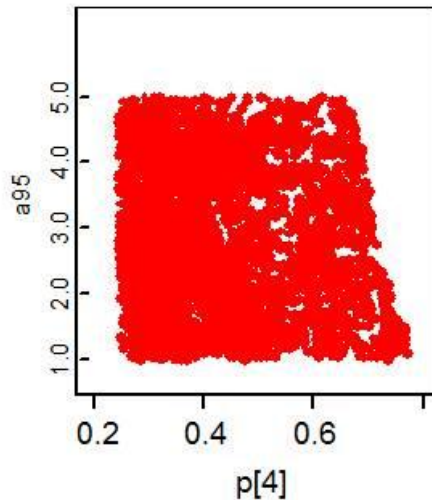
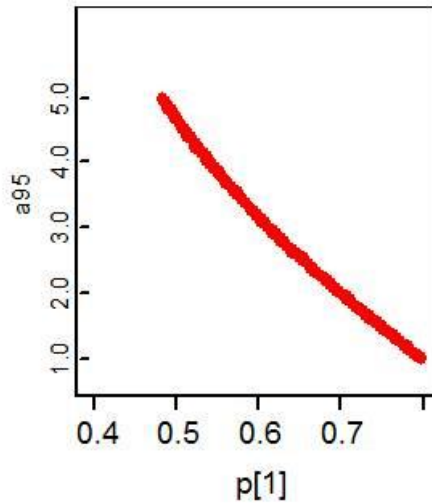


← Vague prior distribution for function parameters instead of point values – identifiability?

Sensitivity to parameter assumptions

Test sensitivity $p[i]$ at
1st and 4th
testing time:
how they depend on
parameters
of the function.

$$p(d) = p^* (1 - e^{-ad}) e^{-a \max\{d - d^{**}\}}$$



95% of max sensitivity ($p^*=0.9$)
achieved in $a_w \sim U(1,5)$ weeks.
 $a_{95} = -\log(0.05)/(a_w/52)$

Decrease begins after
 $d^{**} \sim U(8,16)$ weeks.

- To conclude:
- **Observed data represent the only direct evidence we have about a current situation in a food production chain, e.g. egg production, under risk of e.g. salmonella.**
- **However, information in surveillance data depends on sensitivity of the whole testing scheme which can be variable over time or geographical areas.**
- **This is challenging for inference, because the underlying process state and the sensitivity need to be jointly modeled.**

- To conclude:
- Bayesian inference deals with unknown parameters jointly, instead of an independent analysis for each, which would not represent the combined uncertainty.
- Computationally intensive, so try not to make it more complicated than necessary – “models should be kept as simple as possible, but not more”.

– Merci beaucoup pour votre attention!

A biased selection of some papers:

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- Manuscript under preparation of the hidden Markov model of this talk.